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T. Y. B. Sc. (Sem.-VI) Examination March - 2026

Mathematics

Number Theory - II (OLD) (Paper - MTH - 606/Maths XXII)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book
(2) All questions are compulsory.
(3) Follow usual notations.
(4) Figures to the right indicate marks of the question.

Seat No.:

Student's Signature

Q. 1. Answer ANY FIVE of the following: **10**

1. Solve: $5x \equiv 2 \pmod{26}$.
2. Define a multiplicative function with illustration for multiplicative function f if $f(n) \neq 0$ then prove that $f(1) = 1$.
3. Is the converse of Fermat's theorem true? Justify your answer.
4. For $k \geq 2$, show that $n = 2^{k-1}$ satisfies $\sigma(n) = 2n-1$.
5. If n is an odd integer then prove that $\phi(2n) = \phi(n)$.
6. Find $\phi(36000)$.
7. Find all the terms of positive integer n satisfying $\tau(n) = 30$.
What is the smallest positive integer for which this is true?
8. Find the value of σ and τ for $n = 180$.

Q. 2. Answer ANY TWO questions: **10**

1. Solve the system of linear congruence:
$x \equiv 2 \pmod{3}$
$x \equiv 3 \pmod{5}$
$x \equiv 7 \pmod{7}$
2. Define: Linear congruence. Solve the linear congruence $9x \equiv 21 \pmod{30}$.
3. Find an integer having reminder 1, 2, 3 when divided by 3, 5, 7 respectively.

Q. 3. Answer ANY TWO questions: **10**

1. If p and q are distinct primes such that $a^q \equiv a \pmod{p}$ and $a^p \equiv a \pmod{q}$ then $a^{pq} \equiv a \pmod{pq}$.
2. Find the reminder when 9^{1032} is divided by 13.
3. If $n = 5186$, show that $\phi(n) = \phi(n+1) = \phi(n+2)$.

Q. 4. **Answer ANY TWO questions:** **10**

1. Define Mobius function. For any positive integer n, show that $\mu(n)\,\mu(n+1)\,\mu(n+2)\,\mu(n+3)=0$.
2. Prove that $\tau(n)$ is an odd integer if and only if n is a perfect square.
3. Find the number of zeros in which 100! terminates.

Q. 5. **Answer ANY TWO questions:** **10**

1. If n is apositive integer and gcd $(a, n) = 1$, then prove that $a^{\phi(n)} \equiv 1 \pmod{n}$.
2. If P is a prime and $k \geq 2$ then show that $\phi(\phi(P^{k})) = p^{k-2}\ \phi((p-1)^2)$.
3. Prove that $\phi(3n) = 3\phi(n)$ ifand only if $3 \mid n$.